3235 Probability Theory 3/7/23

In a drawer you have 15 spoons. Pick 3 at random and then put them back.

a) Prob that a given spoon is picked up on a given day:

$$p_1 = \frac{3}{15} = \frac{1}{5}$$

Never picked up

$$p = \left(1 - \frac{1}{5}\right)^5 = \left(\frac{4}{5}\right)^5$$

b)

$$q = 1 - \frac{15!}{(15 \cdot 14 \cdot 13)^5} = 1 - \frac{12!}{(15 \cdot 14 \cdot 13)^4}$$

$$q_i = 1 - \frac{15!}{(3!)^5} \Big/ \binom{15}{3}^5 \qquad \binom{15}{3} = \frac{15 \cdot 14 \cdot 13}{3!}$$

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$$Z = \max_{i=1..N} X_i$$

$$T = \min_{i=1..N} X_i$$

$$\mathbb{P}(Z = x_k \ \& \ T = x_0) =$$

$$1 - \mathbb{P}(Z < x_k \text{ or } T > x_0) =$$

$$1 - \mathbb{P}(Z < x_k) - \mathbb{P}(T > x_0) +$$

$$\mathbb{P}(Z < x_k \ \& \ T > x_0) \geq$$

$$1 - \mathbb{P}(Z < x_k) - \mathbb{P}(T > x_0)$$

$$\mathbb{P}(Z < x_k) = \mathbb{P}(\max_{X_i} < x_k) =$$

$$\mathbb{P}(\text{all } X_i < x_k) =$$

$$\left(\mathbb{P}(X_1 < x_k)\right)^N =$$

$$\left(1 - \mathbb{P}(X_1 = x_k)\right)^N \xrightarrow[N \to \infty]{} 0$$

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p infection rate

1 prob $(1-p)^K$

$K+1$ $1-(1-p)^K$

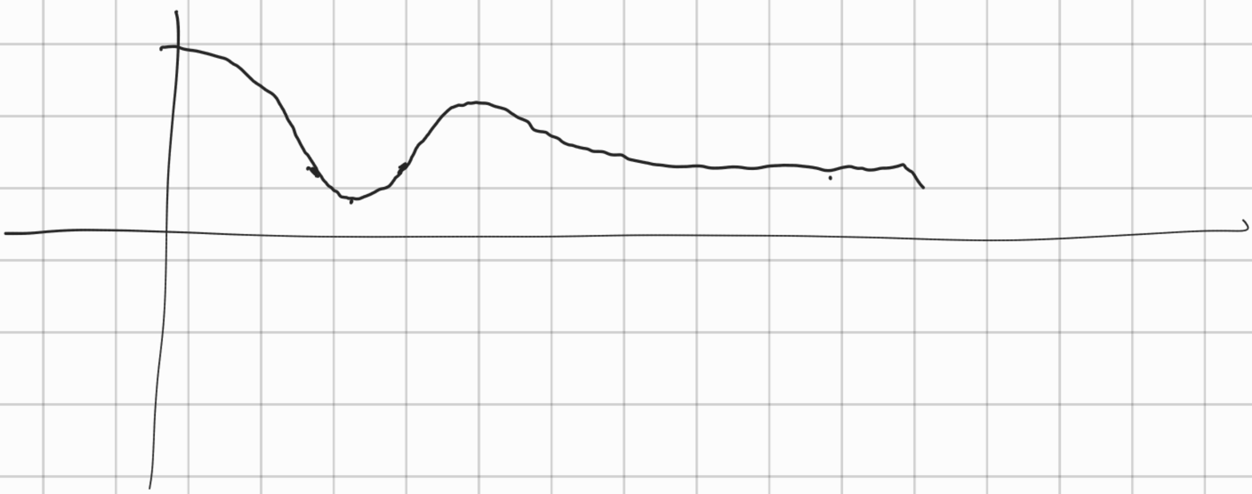
group of K

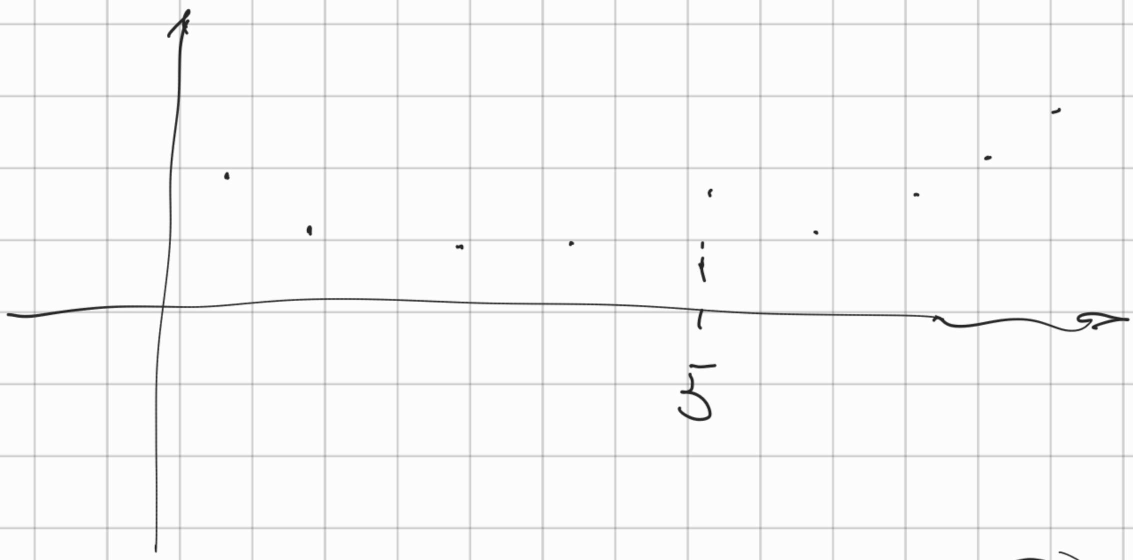
$$q = (1-p)$$

$$\varepsilon = q^K + (K+1)(1-q^K) = 1 + K(1-q^K)$$

$$E = N\left(\frac{1}{K} + (1-q^K)\right)$$

$f(K)$ $f'(K) = 0$





Production line for computers.

$p = 0.05$ of a computer being defective.

Quality control:

defective $\Rightarrow$ discarded prob 1

not defective $\Rightarrow$ discarded prob 0.03

$A = \{ \text{defective} \}$

$B = \{ \text{discarded} \}$

$$\mathbb{P}(B) = \mathbb{P}(B \mid A)\,\mathbb{P}(A) + \mathbb{P}(B \mid A^c)\,\mathbb{P}(A^c)$$

$$= 1 \cdot 0.05 + 0.03 \cdot 0.95 = 0.079$$

$$\mathbb{P}(A|B) = \mathbb{P}(B|A)\frac{\mathbb{P}(A)}{\mathbb{P}(B)} = 0.64$$

Y number of discarded computer That are working among 1000 computer.

$$\mathbb{P}(A^c \cap B) = \mathbb{P}(A^c|B)\,\mathbb{P}(B) = 0.95 \cdot 0.03 =$$
$$= 0.0285$$

$$\mathbb{E}(Y) = 1000 \cdot 0.0285 = 28.5$$

$$\mathbb{P}(Y = 20) = \binom{1000}{20} 0.9715^{980}\, 0.0285^{20}$$

$$\simeq e^{-28.5}\frac{28.5^{20}}{20!}$$

$$\operatorname{Cov}(X_1, X_2) = \mathbb{E}(X_1 X_2) - \mathbb{E}(X_1)\mathbb{E}(X_2)$$

$$\mathbb{E}(X_1) = 1 \cdot \mathbb{P}(X_1 = 1) + 0 \cdot \mathbb{P}(X_1 = 0) =$$

$$= \mathbb{P}(X_1 = 1)$$

$$\mathbb{E}(X_2) = \mathbb{P}(X_2 = 1)$$

$$\mathbb{E}(X_1 X_2) = 1 \cdot 1 \cdot \mathbb{P}(X_1 = 1 \;\&\; X_2 = 1) +$$

$$1 \cdot 0 \; \mathbb{P}(X_1 = 1 \;\&\; X_2 = 0) + \dots$$

$$= \mathbb{P}(X_1 = 1 \;\&\; X_2 = 1)$$

$$\operatorname{Cov}(X_1, X_2) = \mathbb{P}(X_1 = 1 \;\&\; X_2 = 1) - \mathbb{P}(X_1 = 1)\mathbb{P}(X_2 = 1)$$

$$\operatorname{Cov}(X_1, X_2) = 0 \implies$$

$$p_{X_1}(1)\, p_{X_2}(1) = p_{X_1, X_2}(1, 1)$$

$$\{X_1 = 1\} \perp\!\!\!\perp \{X_2 = 1\}$$

$$\{X_1 = 1\} \perp\!\!\!\perp \{X_2 = 1\}^c = \{X_2 = 0\}$$

$$\{X_1 = 0\} \perp\!\!\!\perp \{X_2 = 1\}$$

$$\{X_1 = 0\} \perp\!\!\!\perp \{X_2 = 0\}$$

0

$X_1 \qquad X_2$

$$\begin{array}{ccc} & -1 & 1 \\ P = & \frac{1}{2} & \frac{1}{2} \end{array}$$

$$X_3 = X_1 X_2$$

$$\mathbb{E}(X_1 X_3) = \mathbb{E}(X_1^2 X_2) = \mathbb{E}(X_1^2)\,\mathbb{E}(X_2) = 0$$

$$\mathbb{E}(X_2 X_3) = 0$$

$$\mathbb{E}(X_1 X_2) = 0$$

$$Y_i = \frac{X_1 + 1}{2}$$ Bernoulli.